3.4 Measures of Position and Outliers

Objectives

1. Determine and interpret z-scores
2. Interpret percentiles
3. Determine and interpret quartiles
4. Determine and interpret the interquartile range
5. Check a set of data for outliers

In Section 3.1, we determined measures of central tendency, which describe the *typical* data value. Section 3.2 discussed measures of dispersion, which describe the amount of *spread* in a set of data. In this section, we discuss measures of position, which describe the *relative position* of a certain data value within the entire set of data.

1 Determine and Interpret z-Scores

At the end of the 2014 season, the Los Angeles Angels led the American League with 773 runs scored, while the Colorado Rockies led the National League with 755 runs scored. It appears that the Angels are the better run-producing team. However, this comparison is unfair because the two teams play in different leagues. The Angels play in the American League, where the designated hitter bats for the pitcher, whereas the Rockies play in the National League, where the pitcher must bat (pitchers are typically poor hitters). To compare the two teams' scoring of runs, we need to determine their relative standings in their respective leagues. We can do this using a *z-score*.

Definition

The ***z*-score** represents the distance that a data value is from the mean in terms of the number of standard deviations. We find it by subtracting the mean from the data value and dividing this result by the standard deviation. There is both a population z-score and a sample z-score:

Population z-Score	Sample z-Score	
$z = \dfrac{x - \mu}{\sigma}$	$z = \dfrac{x - \bar{x}}{s}$	**(1)**

The z-score is unitless. It has mean 0 and standard deviation 1.

In Other Words
The z-score provides a way to compare apples to oranges by converting variables with different centers or spreads to variables with the same center (0) and spread (1).

If a data value is larger than the mean, the z-score is positive. If a data value is smaller than the mean, the z-score is negative. If the data value equals the mean, the z-score is zero. A z-score measures the number of standard deviations an observation is above or below the mean. For example, a z-score of 1.24 means the data value is 1.24 standard deviations above the mean. A z-score of -2.31 means the data value is 2.31 standard deviations below the mean.

EXAMPLE 1 Comparing z-Scores

Problem Determine whether the Los Angeles Angels or the Colorado Rockies had a relatively better run-producing season. The Angels scored 773 runs and play in the American League, where the mean number of runs scored was $\mu = 677.4$ and the standard deviation was $\sigma = 51.7$ runs. The Rockies scored 755 runs and play in the National League, where the mean number of runs scored was $\mu = 640.0$ and the standard deviation was $\sigma = 55.9$ runs.

Approach To determine which team had the relatively better run-producing season, compute each team's z-score. The team with the higher z-score had the better season. Because we know the values of the population parameters, compute the population z-score.

Solution We compute each team's z-score, rounded to two decimal places.

Angels: $z\text{-score} = \dfrac{x - \mu}{\sigma} = \dfrac{773 - 677.4}{51.7} = 1.85$

Rockies: $z\text{-score} = \dfrac{x - \mu}{\sigma} = \dfrac{755 - 640.0}{55.9} = 2.06$

So the Angels had run production 1.85 standard deviations above the mean, while the Rockies had run production 2.06 standard deviations above the mean. Therefore, the Rockies had a relatively better year at scoring runs than the Angels. ●

● **Now Work Problem 5**

In Example 1, the team with the higher z-score was said to have a relatively better season in producing runs. With negative z-scores, we need to be careful when deciding the better outcome. For example, suppose Bob and Mary run a marathon. If Bob finished the marathon in 213 minutes, where the mean finishing time among all men was 242 minutes with a standard deviation of 57 minutes, and Mary finished the marathon in 241 minutes, where the mean finishing time among all women was 273 minutes with a standard deviation of 52 minutes, who did better in the race? Since Bob's z-score is $z_{Bob} = \frac{213 - 242}{57} = -0.51$ and Mary's z-score is $z_{Mary} = \frac{241 - 273}{52} = -0.62$, Mary did better. Even though Bob's z-score is larger, Mary did better because she is more standard deviations below the mean.

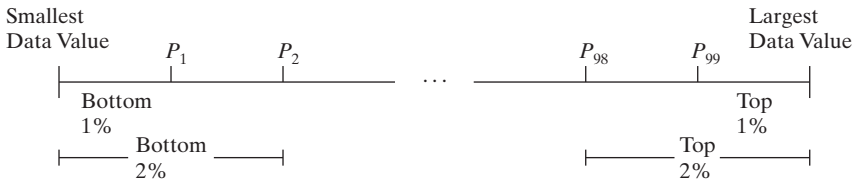
2 Interpret Percentiles

Recall that the median divides the lower 50% of a set of data from the upper 50%. The median is a special case of a general concept called the *percentile*.

Definition The ***k*th percentile**, denoted P_k, of a set of data is a value such that k percent of the observations are less than or equal to the value.

So percentiles divide a set of data that is written in ascending order into 100 parts; thus 99 percentiles can be determined. For example, P_1 divides the bottom 1% of the observations from the top 99%, P_2 divides the bottom 2% of the observations from the top 98%, and so on. Figure 17 displays the 99 possible percentiles.

Figure 17



Percentiles are used to give the relative standing of an observation. Many standardized exams, such as the SAT college entrance exam, use percentiles to let students know how they scored on the exam in relation to all other students who took the exam.

EXAMPLE 2 Interpret a Percentile

Problem Jennifer just received the results of her SAT exam. Her SAT Mathematics score of 600 is in the 74th percentile. What does this mean?

Approach The kth percentile of an observation means that k percent of the observations are less than or equal to the observation.

Interpretation A percentile rank of 74% means that 74% of SAT Mathematics scores are less than or equal to 600 and 26% of the scores are greater. So 26% of the students who took the exam scored better than Jennifer. ●

● Now Work Problem 15

3 Determine and Interpret Quartiles

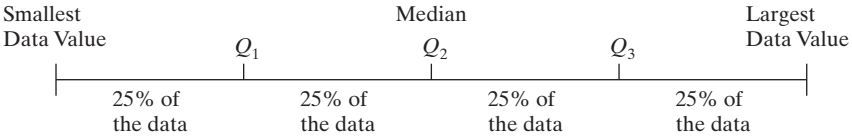
The most common percentiles are quartiles. **Quartiles** divide data sets into fourths, or four equal parts.

In Other Words
The first quartile, Q_1, is equivalent to the 25th percentile, P_{25}. The 2nd quartile, Q_2, is equivalent to the 50th percentile, P_{50}, which is equivalent to the median, M. Finally, the third quartile, Q_3, is equivalent to the 75th percentile, P_{75}.

- The first quartile, denoted Q_1, divides the bottom 25% of the data from the top 75%. Therefore, the first quartile is equivalent to the 25th percentile.
- The second quartile, Q_2, divides the bottom 50% of the data from the top 50%; it is equivalent to the 50th percentile or the median.
- The third quartile, Q_3, divides the bottom 75% of the data from the top 25%; it is equivalent to the 75th percentile.

Figure 18 illustrates the concept of quartiles.

Figure 18



In Other Words
To find Q_2, determine the median of the data set. To find Q_1, determine the median of the "lower half" of the data set. To find Q_3, determine the median of the "upper half" of the data set.

Finding Quartiles

Step 1 Arrange the data in ascending order.

Step 2 Determine the median, M, or second quartile, Q_2.

Step 3 Divide the data set into halves: the observations below (to the left of) M and the observations above M. The first quartile, Q_1, is the median of the bottom half of the data and the third quartile, Q_3, is the median of the top half of the data.

EXAMPLE 3 Finding and Interpreting Quartiles

Problem The Highway Loss Data Institute routinely collects data on collision coverage claims. Collision coverage insures against physical damage to an insured individual's vehicle. The data in Table 16 represent a random sample of 18 collision coverage claims based on data obtained from the Highway Loss Data Institute. Find and interpret the first, second, and third quartiles for collision coverage claims.

Table 16

$6751	$9908	$3461	$2336	$21,147	$2332
$189	$1185	$370	$1414	$4668	$1953
$10,034	$735	$802	$618	$180	$1657

Approach Follow the steps given above.

Solution

Step 1 The data written in ascending order are given as follows:

$180	$189	$370	$618	$735	$802	$1185	$1414	$1657
$1953	$2332	$2336	$3461	$4668	$6751	$9908	$10,034	$21,147

Step 2 There are $n = 18$ observations, so the median, or second quartile, Q_2, is the mean of the 9th and 10th observations. Therefore, $M = Q_2 = \frac{\$1657 + \$1953}{2} = \$1805$.

Step 3 The median of the bottom half of the data is the first quartile, Q_1. As shown next, the median of these data is the 5th observation, so $Q_1 = \$735$.

$$\$180 \quad \$189 \quad \$370 \quad \$618 \quad \underset{\uparrow}{\$735} \quad \$802 \quad \$1185 \quad \$1414 \quad \$1657$$
$$Q_1$$

NOTE
If the number of observations is odd, do not include the median when determining Q_1 and Q_3 by hand. •

(continued)

The median of the top half of the data is the third quartile, Q_3. As shown next, the median of these data is the 5th observation, so $Q_3 = \$4668$.

$$\$1953 \quad \$2332 \quad \$2336 \quad \$3461 \quad \$4668 \quad \$6751 \quad \$9908 \quad \$10{,}034 \quad \$21{,}147$$
$$\uparrow$$
$$Q_3$$

Interpretation Interpret the quartiles as percentiles. For example, 25% of the collision claims are less than or equal to the first quartile, \$735, and 75% of the collision claims are greater than \$735. Also, 50% of the collision claims are less than or equal to \$1805, the second quartile, and 50% of the collision claims are greater than \$1805. Finally, 75% of the collision claims are less than or equal to \$4668, the third quartile, and 25% of the collision claims are greater than \$4668. ●

EXAMPLE 4 Finding Quartiles Using Technology

Problem Find the quartiles of the collision coverage claims data in Table 16.

Using Technology
Statistical packages may use different formulas for obtaining the quartiles, so results may differ slightly.

Approach Use both StatCrunch and Minitab to obtain the quartiles. The steps for obtaining quartiles using a TI-83/84 Plus graphing calculator, Minitab, Excel, and StatCrunch are given in the Technology Step-by-Step on pages 160–161.

Solution The results obtained from StatCrunch [Figure 19(a)] agree with our "by hand" solution. In Figure 19(b), notice that the first quartile, 706, and the third quartile, 5189, reported by Minitab disagree with our "by hand" and StatCrunch result. This difference is due to the fact that StatCrunch and Minitab use different algorithms for obtaining quartiles.

Figure 19

Summary statistics:

Column	n	Median	Min	Max	Q1	Q3
Claim	18	1805	180	21147	735	4668

(a)

Descriptive statistics: Claim

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Claim	18	0	3874	1250	5302	180	706	1805	5189	21447

(b)

● Now Work Problem 21(b) ●

4 Determine and Interpret the Interquartile Range

So far we have discussed three measures of dispersion: range, standard deviation, and variance, all of which are not resistant. Quartiles, however, are resistant. For this reason, quartiles are used to define a fourth measure of dispersion.

Definition

The **interquartile range**, **IQR**, is the range of the middle 50% of the observations in a data set. That is, the IQR is the difference between the third and first quartiles and is found using the formula

$$\text{IQR} = Q_3 - Q_1$$

The interpretation of the interquartile range is similar to that of the range and standard deviation. That is, the more spread a set of data has, the higher the interquartile range will be.

EXAMPLE 5 Determining and Interpreting the Interquartile Range

Problem Determine and interpret the interquartile range of the collision claim data from Example 3.

Approach Use the quartiles found by hand in Example 3. The interquartile range, IQR, is found by computing the difference between the third and first quartiles. It represents the range of the middle 50% of the observations.

Solution The interquartile range is

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= \$4668 - \$735 \\ &= \$3933 \end{aligned}$$

Interpretation The IQR, that is, the range of the middle 50% of the observations, in the collision claim data is \$3933. ●

● Now Work Problem 21(c)

Let's compare the measures of central tendency and dispersion discussed thus far for the collision claim data. The mean collision claim is \$3874.4 and the median is \$1805. The median is more representative of the "center" because the data are skewed to the right (only 5 of the 18 observations are greater than the mean). The range is \$21,147 − \$180 = \$20,967. The standard deviation is \$5301.6 and the interquartile range is \$3933. The values of the range and standard deviation are affected by the extreme claim of \$21,147. In fact, if this claim had been \$120,000 (let's say the claim was for a totaled Mercedes S-class AMG), then the range and standard deviation would increase to \$119,820 and \$27,782.5, respectively. The interquartile range would not be affected. Therefore, when the distribution of data is highly skewed or contains extreme observations, it is best to use the interquartile range as the measure of dispersion because it is resistant.

Summary: Which Measures to Report

Shape of Distribution	Measure of Central Tendency	Measure of Dispersion
Symmetric	Mean	Standard deviation
Skewed left or skewed right	Median	Interquartile range

For the remainder of this text, the direction **describe the distribution** will mean to describe its shape (skewed left, skewed right, symmetric), its center (mean or median), and its spread (standard deviation or interquartile range).

5 Check a Set of Data for Outliers

CAUTION! Outliers distort both the mean and the standard deviation, because neither is resistant. Because these measures often form the basis for most statistical inference, any conclusions drawn from a set of data that contains outliers can be flawed.

When performing any type of data analysis, we should always check for extreme observations in the data set. Extreme observations are referred to as **outliers**. Outliers can occur by chance, because of error in the measurement of a variable, during data entry, or from errors in sampling. For example, in the 2000 presidential election, a precinct in New Mexico accidentally recorded 610 absentee ballots for Al Gore as 110. Workers in the Gore camp discovered the data-entry error through an analysis of vote totals.

Outliers do not always occur because of error. Sometimes extreme observations are common within a population. For example, suppose we wanted to estimate the mean price of a European car. We might take a random sample of size 5 from the population of all European automobiles. If our sample included a Ferrari F430 Spider

(approximately \$175,000), it probably would be an outlier, because this car costs much more than the typical European automobile. The value of this car would be considered *unusual* because it is not a typical value from the data set.

Use the following steps to check for outliers using quartiles.

Checking for Outliers by Using Quartiles

Step 1 Determine the first and third quartiles of the data.

Step 2 Compute the interquartile range.

Step 3 Determine the fences. **Fences** serve as cutoff points for determining outliers.

$$\text{Lower fence} = Q_1 - 1.5(\text{IQR})$$
$$\text{Upper fence} = Q_3 + 1.5(\text{IQR})$$

Step 4 If a data value is less than the lower fence or greater than the upper fence, it is considered an outlier.

EXAMPLE 6 Checking for Outliers

Problem Check the collision coverage claims data in Table 16 for outliers.

Approach Follow the preceding steps. Any data value that is less than the lower fence or greater than the upper fence will be considered an outlier.

Solution

Step 1 The quartiles found in Example 3 are $Q_1 = \$735$ and $Q_3 = \$4668$.

Step 2 The interquartile range, IQR, is

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1\\ &= \$4668 - \$735\\ &= \$3933\end{aligned}$$

Step 3 The lower fence, LF, is

$$\begin{aligned}\text{LF} &= Q_1 - 1.5(\text{IQR})\\ &= \$735 - 1.5(\$3933)\\ &= -\$5164.5\end{aligned}$$

The upper fence, UF, is

$$\begin{aligned}\text{UF} &= Q_3 + 1.5(\text{IQR})\\ &= \$4668 + 1.5(\$3933)\\ &= \$10{,}567.5\end{aligned}$$

Step 4 There are no observations below the lower fence. However, there is an observation above the upper fence. The claim of \$21,147 is an outlier. ●

● Now Work Problem 21(d)

Technology Step-by-Step Determining Quartiles

TI-83/84 Plus
Follow the same steps given to compute the mean and median from raw data. (Section 3.1)

Minitab
Follow the same steps given to compute the mean and median from raw data. (Section 3.1)

Excel

1. Enter the raw data into column A.
2. With the data analysis Tool Pak enabled, select the Data tab and click on **Data Analysis**.
3. Select **Rank and Percentile** from the Data Analysis window. Press OK.
4. With the cursor in the **Input Range** cell, highlight the data. Press OK.

StatCrunch

Follow the same steps given to compute the mean and median from raw data. (Section 3.1)

3.4 Assess Your Understanding

Vocabulary

1. The _____ represents the number of standard deviations an observation is from the mean.

2. The _____ _____ of a data set is a value such that k percent of the observations are less than or equal to the value.

3. _____ divide data sets into fourths.

4. The _____ _____ is the range of the middle 50% of the observations in a data set.

Applying the Concepts

NW **5. Birth Weights** Babies born after a gestation period of 32–35 weeks have a mean weight of 2600 grams and a standard deviation of 660 grams. Babies born after a gestation period of 40 weeks have a mean weight of 3500 grams and a standard deviation of 470 grams. Suppose a 34-week gestation period baby weighs 2400 grams and a 40-week gestation period baby weighs 3300 grams. What is the z-score for the 34-week gestation period baby? What is the z-score for the 40-week gestation period baby? Which baby weighs less relative to the gestation period?

6. Birth Weights Babies born after a gestation period of 32–35 weeks have a mean weight of 2600 grams and a standard deviation of 660 grams. Babies born after a gestation period of 40 weeks have a mean weight of 3500 grams and a standard deviation of 470 grams. Suppose a 34-week gestation period baby weighs 3000 grams and a 40-week gestation period baby weighs 3900 grams. What is the z-score for the 34-week gestation period baby? What is the z-score for the 40-week gestation period baby? Which baby weighs less relative to the gestation period?

7. Men versus Women The average 20- to 29-year-old man is 69.6 inches tall, with a standard deviation of 3.0 inches, while the average 20- to 29-year-old woman is 64.1 inches tall, with a standard deviation of 3.8 inches. Who is relatively taller, a 75-inch man or a 70-inch woman?

Source: CDC Vital and Health Statistics, Advance Data, Number 361, July 5, 2005

8. Men versus Women The average 20- to 29-year-old man is 69.6 inches tall, with a standard deviation of 3.0 inches, while the average 20- to 29-year-old woman is 64.1 inches tall, with a standard deviation of 3.8 inches. Who is relatively taller, a 67-inch man or a 62-inch woman?

Source: CDC Vital and Health Statistics, Advance Data, Number 361, July 5, 2005

9. ERA Champions In 2014, Clayton Kershaw of the Los Angeles Dodgers had the lowest earned-run average (ERA is the mean number of runs yielded per nine innings pitched) of any starting pitcher in the National League, with an ERA of 1.77. Also in 2014, Felix Hernandez of the Seattle Mariners had the lowest ERA of any starting pitcher in the American League with an ERA of 2.14. In the National League, the mean ERA in 2014 was 3.430 and the standard deviation was 0.721. In the American League, the mean ERA in 2014 was 3.598 and the standard deviation was 0.762. Which player had the better year relative to his peers? Why?

10. Batting Champions The highest batting average ever recorded in Major League Baseball was by Ted Williams in 1941 when he hit 0.406. That year, the mean and standard deviation for batting average were 0.2806 and 0.0328. In 2014, Jose Altuve was the American League batting champion, with a batting average of 0.341. In 2014, the mean and standard deviation for batting average were 0.2679 and 0.0282. Who had the better year relative to his peers, Williams or Altuve? Why?

11. Swim Ryan Murphy, nephew of the author, swims for the University of California at Berkeley. Ryan's best time in the 100-meter backstroke is 45.3 seconds. The mean of all NCAA swimmers in this event is 48.62 seconds with a standard deviation of 0.98 second. Ryan's best time in the 200-meter backstroke is 99.32 seconds. The mean of all NCAA swimmers in this event is 106.58 seconds with a standard deviation of 2.38 seconds. In which race is Ryan better?

12. Triathlon Roberto finishes a triathlon (750-meter swim, 5-kilometer run, and 20-kilometer bicycle) in 63.2 minutes. Among all men in the race, the mean finishing time was 69.4 minutes with a standard deviation of 8.9 minutes. Zandra finishes the same triathlon in 79.3 minutes. Among all women in the race, the mean finishing time was 84.7 minutes with a standard deviation of 7.4 minutes. Who did better in relation to their gender?

13. School Admissions A highly selective boarding school will only admit students who place at least 1.5 standard deviations above the mean on a standardized test that has a mean of 200 and a standard deviation of 26. What is the minimum score that an applicant must make on the test to be accepted?

14. Quality Control A manufacturer of bolts has a quality-control policy that requires it to destroy any bolts that are more than 2 standard deviations from the mean. The quality-control engineer knows that the bolts coming off the assembly line have

a mean length of 8 cm with a standard deviation of 0.05 cm. For what lengths will a bolt be destroyed?

NW **15. You Explain It! Percentiles** Explain the meaning of the following percentiles.

Source: Advance Data from Vital and Health Statistics

(a) The 15th percentile of the head circumference of males 3 to 5 months of age is 41.0 cm.

(b) The 90th percentile of the waist circumference of females 2 years of age is 52.7 cm.

(c) Anthropometry involves the measurement of the human body. One goal of these measurements is to assess how body measurements may be changing over time. The following table represents the standing height of males aged 20 years or older for various age groups. Based on the percentile measurements of the different age groups, what might you conclude?

	Percentile				
Age	**10th**	**25th**	**50th**	**75th**	**90th**
20–29	166.8	171.5	176.7	181.4	186.8
30–39	166.9	171.3	176.0	181.9	186.2
40–49	167.9	172.1	176.9	182.1	186.0
50–59	166.0	170.8	176.0	181.2	185.4
60–69	165.3	170.1	175.1	179.5	183.7
70–79	163.2	167.5	172.9	178.1	181.7
80 or older	161.7	166.1	170.5	175.3	179.4

16. You Explain It! Percentiles Explain the meaning of the following percentiles.

Source: National Center for Health Statistics.

(a) The 5th percentile of the weight of males 36 months of age is 12.0 kg.

(b) The 95th percentile of the length of newborn females is 53.8 cm.

17. You Explain It! Quartiles Violent crimes include rape, robbery, assault, and homicide. The following is a summary of the violent-crime rate (violent crimes per 100,000 population) for all 50 states in the United States plus Washington, D.C., in 2012.

$$Q_1 = 252.4 \quad Q_2 = 333.8 \quad Q_3 = 454.5$$

(a) Provide an interpretation of these results.

(b) Determine and interpret the interquartile range.

(c) The violent-crime rate in Washington, D.C., in 2012 was 1243.7. Would this be an outlier?

(d) Do you believe that the distribution of violent-crime rates is skewed or symmetric? Why?

18. You Explain It! Quartiles One variable that is measured by online homework systems is the amount of time a student spends on homework for each section of the text. The following is a summary of the number of minutes a student spends for each section of the text for the fall 2014 semester in a College Algebra class at Joliet Junior College.

$$Q_1 = 42 \quad Q_2 = 51.5 \quad Q_3 = 72.5$$

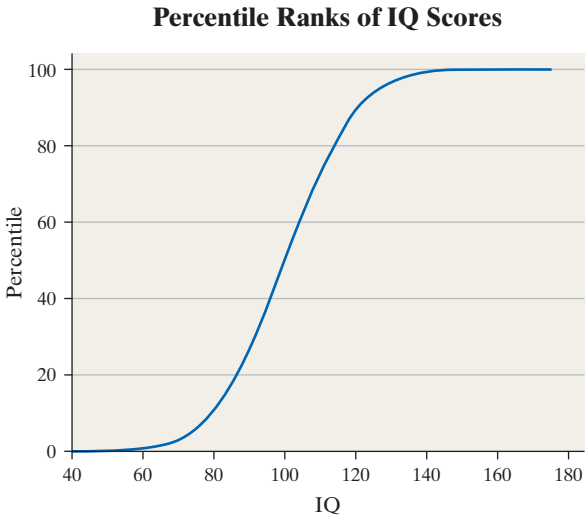
(a) Provide an interpretation of these results.

(b) Determine and interpret the interquartile range.

(c) Suppose a student spent 2 hours doing homework for a section. Is this an outlier?

(d) Do you believe that the distribution of time spent doing homework is skewed or symmetric? Why?

19. Ogives and Percentiles The following graph is an ogive of IQ scores. The vertical axis in an ogive is the cumulative relative frequency and can also be interpreted as a percentile.

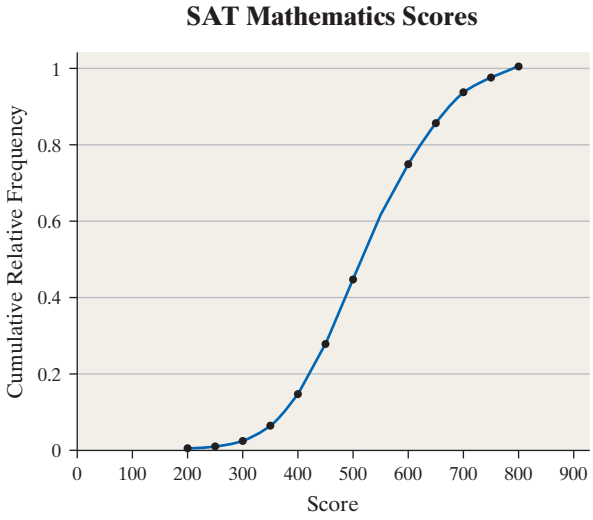


(a) Find and interpret the percentile rank of an individual whose IQ is 100.

(b) Find and interpret the percentile rank of an individual whose IQ is 120.

(c) What score corresponds to the 60th percentile for IQ?

20. Ogives and Percentiles The following graph is an ogive of the mathematics scores on the SAT. The vertical axis in an ogive is the cumulative relative frequency and can also be interpreted as a percentile.



(a) Find and interpret the percentile rank of a student who scored 450 on the SAT mathematics exam.

(b) Find and interpret the percentile rank of a student who scored 750 on the SAT mathematics exam.

(c) If Jane scored at the 44th percentile, what was her score?

NW **21. SMART Car** The following data represent the miles per gallon of a random sample of SMART cars with a three-cylinder, 1.0-liter engine.

31.5	36.0	37.8	38.4	40.1	42.3
34.3	36.3	37.9	38.8	40.6	42.7
34.5	37.4	38.0	39.3	41.4	43.5
35.5	37.5	38.3	39.5	41.5	47.5

Source: www.fueleconomy.gov

(a) Compute the z-score corresponding to the individual who obtained 36.3 miles per gallon. Interpret this result.
(b) Determine the quartiles.
(c) Compute and interpret the interquartile range, IQR.
(d) Determine the lower and upper fences. Are there any outliers?

22. Hemoglobin in Cats The following data represent the hemoglobin (in g/dL) for 20 randomly selected cats.

5.7	8.9	9.6	10.6	11.7
7.7	9.4	9.9	10.7	12.9
7.8	9.5	10.0	11.0	13.0
8.7	9.6	10.3	11.2	13.4

Source: Joliet Junior College Veterinarian Technology Program

(a) Compute the z-score corresponding to the hemoglobin of Blackie, 7.8 g/dL. Interpret this result.
(b) Determine the quartiles.
(c) Compute and interpret the interquartile range, IQR.
(d) Determine the lower and upper fences. Are there any outliers?

23. Rate of Return of Google The following data represent the monthly rate of return of Google common stock from its inception in January 2007 through November 2014.

−0.10	−0.02	0.00	0.02	−0.10	0.03	0.04	−0.15	−0.08
0.02	0.01	−0.18	−0.10	−0.18	0.14	0.07	−0.01	0.09
0.03	0.10	−0.17	−0.10	0.05	0.05	0.08	0.08	−0.07
0.06	0.25	−0.07	−0.02	0.10	0.01	0.09	−0.07	0.17
0.05	−0.02	0.30	−0.14	0.00	0.05	0.06	−0.08	0.17

Source: Yahoo!Finance

(a) Determine and interpret the quartiles.
(b) Check the data set for outliers.

24. CO_2 Emissions The following data represent the carbon dioxide emissions from the consumption of energy per capita (total carbon dioxide emissions, in tons, divided by total population) for the countries of Europe.

1.31	5.38	10.36	5.73	3.57	5.40	6.24
8.59	9.46	6.48	11.06	7.94	4.63	6.12
14.87	9.94	10.06	10.71	15.86	6.93	3.58
4.09	9.91	161.57	7.82	8.70	8.33	9.38
7.31	16.75	9.95	23.87	7.76	8.86	

Source: Carbon Dioxide Information Analysis Center

(a) Determine and interpret the quartiles.
(b) Is the observation corresponding to Albania, 1.31, an outlier?

25. Fraud Detection As part of its "Customers First" program, a cellular phone company monitors monthly phone usage. The program identifies unusual use and alerts the customer that their phone may have been used by another person. The data below represent the monthly phone use in minutes of a customer enrolled in this program for the past 20 months. The phone company decides to use the upper fence as the cutoff point for the number of minutes at which the customer should be contacted. What is the cutoff point?

346	345	489	358	471
442	466	505	466	372
442	461	515	549	437
480	490	429	470	516

26. Stolen Credit Card A credit card company has a fraud-detection service that determines if a card has any unusual activity. The company maintains a database of daily charges on a customer's credit card. Days when the card was inactive are excluded from the database. If a day's worth of charges appears unusual, the customer is contacted to make sure that the credit card has not been compromised. Use the following daily charges (rounded to the nearest dollar) to determine the amount the daily charges must exceed before the customer is contacted.

143	166	113	188	133
90	89	98	95	112
111	79	46	20	112
70	174	68	101	212

27. Student Survey of Income A survey of 50 randomly selected full-time Joliet Junior College students was conducted during the Fall 2015 semester. In the survey, the students were asked to disclose their weekly income from employment. If the student did not work, \$0 was entered.

0	262	0	635	0	0	671
244	521	476	100	650	454	95
12,777	567	310	527	0	67	736
83	159	0	547	188	389	300
719	0	367	316	0	0	181
479	0	82	579	289		
375	347	331	281	628		
0	203	149	0	403		

(a) Check the data set for outliers.
(b) Draw a histogram of the data and label the outliers on the histogram.
(c) Provide an explanation for the outliers.

28. Student Survey of Entertainment Spending A survey of 40 randomly selected full-time Joliet Junior College students was conducted in the Fall 2015 semester. In the survey, the students were asked to disclose their weekly spending on entertainment. The results of the survey are as follows:

21	54	64	33	65	32	21	16
22	39	67	54	22	51	26	14
115	7	80	59	20	33	13	36
36	10	12	101	1000	26	38	8
28	28	75	50	27	35	9	48

(a) Check the data set for outliers.
(b) Draw a histogram of the data and label the outliers on the histogram.
(c) Provide an explanation for the outliers.

29. Pulse Rate Use the results of Problem 21 in Section 3.1 and Problem 19 in Section 3.2 to compute the z-scores for all the students. Compute the mean and standard deviation of these z-scores.

30. Travel Time Use the results of Problem 22 in Section 3.1 and Problem 20 in Section 3.2 to compute the z-scores for all the students. Compute the mean and standard deviation of these z-scores.

31. Fraud Detection Revisited Use the fraud-detection data from Problem 25 to do the following.

(a) Determine the standard deviation and interquartile range of the data.
(b) Suppose the month in which the customer used 346 minutes was not actually that customer's phone. That particular month, the customer did not use her phone at all, so 0 minutes were used. How does changing the observation from 346 to 0 affect the standard deviation and interquartile range? What property does this illustrate?

Explaining the Concepts

32. Write a paragraph that explains the meaning of percentiles.

33. Suppose you received the highest score on an exam. Your friend scored the second-highest score, yet you both were in the 99th percentile. How can this be?

34. Morningstar is a mutual fund rating agency. It ranks a fund's performance by using one to five stars. A one-star mutual fund is in the bottom 10% of its investment class; a five-star mutual fund is at the 90th percentile of its investment class. Interpret the meaning of a five-star mutual fund.

35. When outliers are discovered, should they always be removed from the data set before further analysis?

36. Mensa is an organization designed for people of high intelligence. One qualifies for Mensa if one's intelligence is measured at or above the 98th percentile. Explain what this means.

37. Explain the advantage of using z-scores to compare observations from two different data sets.

38. Explain the circumstances for which the interquartile range is the preferred measure of dispersion. What is an advantage that the standard deviation has over the interquartile range?

39. Explain what each quartile represents.