Reflecting the graph of \( y = \sin x \) on the restricted domain, shown in Figure 5(a), across the line \( y = x \) gives the graph of the inverse function, shown in Figure 5(b). Some key points are labeled on the graph. The equation of the inverse of \( y = \sin x \) is found by interchanging \( x \) and \( y \) to get 

\[ x = \sin y. \]

This equation is solved for \( y \) by writing 

\[ y = \sin^{-1} x \] 
(read “inverse sine of \( x \)”).

As Figure 5(b) shows, the domain of \( y = \sin^{-1} x \) is \([-1, 1]\), while the restricted domain of \( y = \sin x \), \([ -\frac{\pi}{2}, \frac{\pi}{2} ]\), is the range of \( y = \sin^{-1} x \). An alternative notation for \( \sin^{-1} x \) is \( \arcsin x \).

**Inverse Sine Function**

\[ y = \sin^{-1} x \text{ or } y = \arcsin x \] means that \( x = \sin y \), for \(-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\).

We can think of \( y = \sin^{-1} x \) or \( y = \arcsin x \) as

“\( y \) is the number (angle) in the interval \([ -\frac{\pi}{2}, \frac{\pi}{2} ]\) whose sine is \( x \).”

Thus, we can write \( y = \sin^{-1} x \) as \( \sin y = x \) to evaluate it. We must pay close attention to the domain and range intervals.
### Example 1 Finding Inverse Sine Values

Find $y$ in each equation.

(a) $y = \arcsin \frac{1}{2}$

**Algebraic Solution**

(a) The graph of the function defined by $y = \arcsin x$ (Figure 5(b)) includes the point $(\frac{1}{2}, \frac{\pi}{6})$. Therefore, $\arcsin \frac{1}{2} = \frac{\pi}{6}$.

Alternatively, we can think of $y = \arcsin \frac{1}{2}$ as “$y$ is the number in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ whose sine is $\frac{1}{2}$.” Then we can write the given equation as $\sin y = \frac{1}{2}$.

Since $\sin \frac{\pi}{6} = \frac{1}{2}$ and $\frac{\pi}{6}$ is in the range of the arcsine function, $y = \frac{\pi}{6}$.

(b) Writing the equation $y = \sin^{-1}(1)$ in the form $\sin y = -1$ shows that $y = -\frac{\pi}{2}$. Notice that the point $(-1, -\frac{\pi}{2})$ is on the graph of $y = \sin^{-1} x$.

(c) Because $-2$ is not in the domain of the inverse sine function, $\sin^{-1}(-2)$ does not exist.

**Graphing Calculator Solution**

We graph the equation $y_1 = \sin^{-1} x$ and find the points with $x$-values $1/2$ and $-1$.

For these two $x$-values, Figure 6 indicates that $Y_1 = \frac{\pi}{6}$ and $Y = -\frac{\pi}{2} \approx -1.570796$.

We summarize this discussion about the inverse sine function as follows.

**Inverse Sine Function** $y = \sin^{-1} x$ or $y = \arcsin x$

- Domain: $[-1, 1]$  
- Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

- The inverse sine function is increasing and continuous on its domain $[-1, 1]$.
- Its $x$-intercept is $0$, and its $y$-intercept is $0$.
- Its graph is symmetric with respect to the origin, so the function is an odd function. For all $x$ in the domain, $\sin^{-1}(-x) = -\sin^{-1} x$. 

### CAUTION

In Example 1(b), it is tempting to give the value of $\sin^{-1}(-1)$ as $\frac{3\pi}{2}$, since $\sin \frac{3\pi}{2} = -1$. Notice, however, that $\frac{3\pi}{2}$ is not in the range of the inverse sine function. **Be certain that the number given for an inverse function value is in the range of the particular inverse function being considered.**
We can think of \( y = \cos^{-1} x \) (or \( y = \arccos x \)) as “\( y \) is the number (angle) in the interval \( [0, \pi] \) whose cosine is \( x \).”

**Inverse Cosine Function**

**Example 2** Finding Inverse Cosine Values

Find \( y \) in each equation.

(a) \( y = \arccos 1 \)

(b) \( y = \cos^{-1} \left(-\frac{\sqrt{2}}{2}\right) \)

**Solution**

(a) Since the point \((1, 0)\) lies on the graph of \( y = \arccos x \) in Figure 9, the value of \( y \), or \( \arccos 1 \), is 0. Alternatively, we can think of \( y = \arccos 1 \) as

“\( y \) is the number in \([0, \pi]\) whose cosine is 1,” or \( \cos y = 1 \).

Thus, \( y = 0 \), since \( \cos 0 = 1 \) and 0 is in the range of the arccosine function.

(b) We must find the value of \( y \) that satisfies

\[
\cos y = -\frac{\sqrt{2}}{2},
\]

where \( y \) is in the interval \([0, \pi]\),

which is the range of the function \( y = \cos^{-1} x \). The only value for \( y \) that satisfies these conditions is \( \frac{3\pi}{4} \). Again, this can be verified from the graph in Figure 9.

\( \checkmark \) Now Try Exercises 15 and 23.
Our observations about the inverse cosine function lead to the following generalizations.

**Inverse Cosine Function**

\( y = \cos^{-1} x \) or \( y = \arccos x \)

Domain: \([-1, 1]\)  
Range: \([0, \pi]\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>(\pi)</td>
</tr>
<tr>
<td>(-\sqrt{3}/2)</td>
<td>3(\pi)/4</td>
</tr>
<tr>
<td>0</td>
<td>(\pi/2)</td>
</tr>
<tr>
<td>(\sqrt{3}/2)</td>
<td>(\pi/4)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 10

- The inverse cosine function is decreasing and continuous on its domain \([-1, 1]\).
- Its \(x\)-intercept is 1, and its \(y\)-intercept is \(\pi\).
- Its graph is not symmetric with respect to either the \(y\)-axis or the origin.

**Inverse Tangent Function**

Restricting the domain of the function \(y = \tan x\) to the open interval \((-\pi/2, \pi/2)\) yields a one-to-one function. By interchanging the roles of \(x\) and \(y\), we obtain the inverse tangent function given by \(y = \tan^{-1} x\) or \(y = \arctan x\). Figure 11 shows the graph of the restricted tangent function. Figure 12 gives the graph of \(y = \tan^{-1} x\).

Figure 11

\(y = \tan^{-1} x\) or \(y = \arctan x\)

**Inverse Tangent Function**

\(y = \tan^{-1} x\) or \(y = \arctan x\) means that \(x = \tan y\), for \(-\pi/2 < y < \pi/2\).

We can think of \(y = \tan^{-1} x\) or \(y = \arctan x\) as

“\(y\) is the number (angle) in the interval \((-\pi/2, \pi/2)\) whose tangent is \(x\).”