# MATH TYPE PRACTICE FILES WITH TRIGONOMETRY

And output to

NVDA with MATH PLAYER

JAWS with MHTML & MATH JAX output

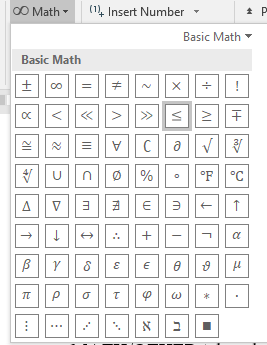
CAR – Central Access Reader with TTS output

DUXBURY Braille (.brf files)

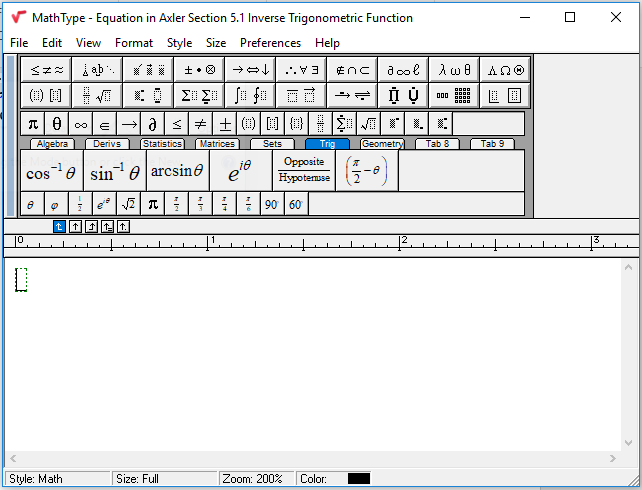
1. Make sure these tools & apps have been downloaded & installed on your computer:
   1. JAWS demo
   2. NVDA
   3. CAT toolbar
   4. CAR reader
   5. Duxbury
   6. MathType
   7. MathPlayer
   8. Chem4Word

This works in WIN OS 10.0 with Microsoft Office 2016

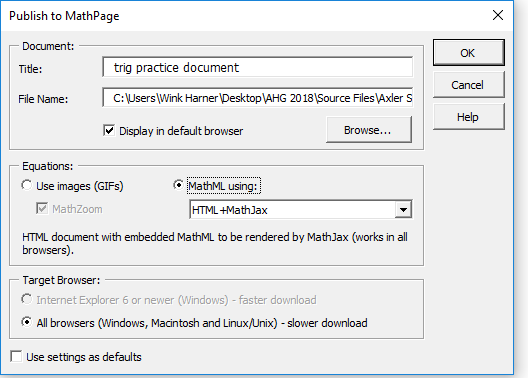
1. Do not open any –none!– of the TTS or Screen Readers. Yet.
2. Create a NEW MSW document, and minimize THIS document and the NEW document so that they fit side by side or top to bottom on a split screen. This allows you to see the TRIG functions which have already been converted from PDF to RTF, run through MATHTYPE and saved as an edited, ready to go source file for trigonometry.
3. You are going to practice typing in and selecting the functions for the figures in this document (below) and testing them out with NVDA and CAR for audio accuracy, setting up and saving as MHTML page for JAWS + MATH JAX output, and exporting with DUXBURY as electronic .brf files.
4. From MATH TYPE toolbar, select INLINE for the equation type input window, then from the MATH/OTHER tab, select MATH and choose BASIC MATH



1. Click on INLINE to open a new MATH TYPE EDITING WINDOW, then Select TRIG from the type of math on the MathType INLINE equation editing window.



1. Type in the MATH TYPE EDITING window 
2. Press the X close window button in MATHTYPE. Say YES to transfer the equation to your MSW Document.
3. Select your newly created trig function, open MATH PLAYER on the MSW toolbar menu. Click on SPEAK icon and listen to the trig function.
4. Continue through this document, and practice retyping the trig functions in MATHTYPE, listening to them in MATH PLAYER.
5. Create a whole list (5 or more along with copying the text that goes along with the problems here) in your new document, SAVE.
6. OPEN with NVDA. Listen.
7. OPEN with Central Access Reader. Listen
8. From MATHTYPE toolbar, select PUBLISH TO MATH PAGE, and from the MATHML using button, select HTML+ MathJax as the output file. Save to your desktop or to a new working folder. OPEN with JAWS. NOTE: you will have to restart your computer in order to use the demo version of JAWS.



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Trigonometric Algebra and Geometry

This chapter begins by introducing the inverse trigonometric functions. These tremendously useful functions allow us to find angles from measurements of lengths. We will pay some attention to the inverse trigonometric identities, which will strengthen our understanding of these functions.

Then we will turn our attention to area, showing how trigonometry can be used to compute areas of various regions. We will also derive some important approximations of the trigonometric functions.

The law of sines and the law of cosines let us use trigonometry to compute all the angles and the lengths of the sides of a triangle given only some of this information. We will see spectacular applications as these results allow us to compute the distance to far-away objects that we cannot physically reach.

The double-angle and half-angle formulas for the trigonometric functions will allow us to compute exact expressions for quantities such as  and  (see Problems 106 and 109 in Section 5.5 for these beautiful expressions). The addition and subtraction formulas for the trigonometric functions provide another group of useful identities.

This chapter concludes with an investigation into transformations of trigonometric functions, which are used to model periodic events. Redoing function transformations in the context of trigonometric functions will also help us review the results from Chapter 1 on how function transformations change graphs.

As usual, we will assume throughout this section that all angles are measured in radians unless explicitly stated otherwise.

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## 5.1 Inverse Trigonometric Functions

Learning Objectives

By the end of this section you should be able to

* compute values of  ,  , and  ;
* sketch the radius of the unit circle corresponding to the arccosine, arcsine, and arctangent of a number;
* use the inverse trigonometric functions to find angles in a right triangle, given the lengths of two sides;
* find the angles in an isosceles triangle, given the lengths of the sides;
* use  to find the angle a line with given slope makes with the horizontal axis.

Several of the most important functions in mathematics are defined as the inverse functions of familiar functions. For example, the cube root is defined as the inverse function of  , and the logarithm base 3 is defined as the inverse function of .

In this section, we will define the inverses of the cosine, sine, and tangent functions. These inverse functions are called the arccosine, the arcsine, and the arctangent. Neither cosine nor sine nor tangent is one-to-one when defined on its usual domain. Thus we will need to restrict the domains of these functions to obtain one-to-one functions that have inverses.

The Arccosine Function

Recall that a function is called one-to-one if it assigns distinct values to distinct numbers in its domain. The cosine function, whose domain is the entire real line, is not one-to-one because, for example, .

Recall also that only one-to-one functions have inverses (see Section 1.5 to review one-to-one functions and their inverses). Thus the cosine function does not have an inverse.

We faced a similar dilemma when we wanted to define the square root function as the inverse of the function . The domain of the function  is the entire real line. This function is not one-to-one; thus it does not have an inverse. We solved this problem by restricting the domain of  to ; the resulting function is one-to-one, and its inverse is called the square root function. Roughly speaking, we say that the square root function is the inverse of .

We will follow a similar process with the cosine. To decide how to restrict the domain of the cosine, we start by declaring that 0 should be in the domain of the restricted function. Looking at the graph above of cosine, we see that starting at 0 and moving to the right,  is the farthest we can go while staying within an interval on which cosine is one-to-one. If  will be in the domain of our restricted cosine function, then we cannot move at all to the left from 0 and still have a one-to-one function. Thus  is the natural domain to choose.

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If we restrict the domain of cosine to , we obtain the one-to-one function whose graph is shown here. The inverse of this function is called the arccosine, which is abbreviated as . Here is the formal definition.

Arccosine

Suppose .

* The arccosine of t, denoted  , is the angle  whose cosine equals t.

Short version:

*  means .

In defining , we must restrict t to be in the interval  because otherwise there is no angle whose cosine equals t.

Example 1

(a) Evaluate .

(b) Evaluate .

(c) Explain why the expression  makes no sense.

Solution

(a) We have , because  and  is in the interval .

(b) We have , because  and 0 is in the interval .

(c) The expression  makes no sense because there is no angle whose cosine equals 2.

Do not confuse  with . Confusion can arise due to inconsistency in common notation. For example,  is indeed equal to . However, we defined  to equal  only when n is a positive integer (see Section 4.6).

This restriction concerning  was made precisely so that  could be defined with  interpreted as an inverse function.

The notation  to denote the arccosine function is consistent with our notation  to denote the inverse of a function f. Even here a bit of explanation helps. The usual domain of the cosine function is the real line. However, when we write  we do not mean the inverse of the usual cosine function (which has no inverse because it is not one-to-one). Instead,  means the inverse of the cosine function whose domain is restricted to the interval .

The three different solutions to the three parts of the next example show why you need to pay careful attention to the meaning of notation.

Example 2

For x = 0.2, evaluate each of the following:

(a) 

(b) 

(c) 

Solution

When checking the results below on your calculator, be sure that it is set to work in radians rather than degrees.

(a)  (note that and 1.36944 is in )

(b) 

(c) 

The next example will help solidify your understanding of the arccosine function.

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Example 3

Sketch the radius of the unit circle corresponding to the angle .

Solution

We seek an angle in  whose cosine equals 0.3. This means that the first coordinate of the endpoint of the corresponding radius will equal 0.3. Thus we start with 0.3 on the horizontal axis, as shown here, and extend a line upward until it intersects the unit circle. That point of intersection is the endpoint of the radius corresponding to the angle  , as shown here (see figure).

A calculator shows that .

Thus the angle  shown in the figure here is approximately 1.266 radians, which is approximately .

Recall that the inverse of a function interchanges the domain and range of the original function. Thus we have the following.

Domain and range of arccosine

* The domain of  is .
* The range of  is .

The graph of  can be obtained in the usual way when dealing with inverse functions. Specifically, flip the graph of the cosine (restricted to the interval ) across the line with slope 1 that contains the origin, getting the graph shown here (see figure).

The inverse trigonometric functions are spectacularly useful in finding the angles of a right triangle when given the lengths of two of the sides. The next example gives our first illustration of this procedure.

Example 4

Suppose a 13-foot ladder is leaned against a building, reaching to the bottom of a second-floor window 12 feet above the ground. What angle does the ladder make with the building?

Solution

Let  denote the angle the ladder makes with the building. Because the cosine of an angle in a right triangle equals the length of the adjacent side divided by the length of the hypotenuse, the figure here shows that .

Thus.

Hence  is approximately 0.3948 radians, which is approximately 22.6°.

The Arcsine Function

Now we consider the sine function, whose graph is shown below (see figure).

If we restrict sine to the orange part of the graph, we get a function that is one-to-one and thus has an inverse.

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We need to restrict the domain to obtain a one-to-one function. We again start by declaring that 0 should be in the domain of the restricted function. Looking at the graph of sine, we see that  is the largest interval containing 0 on which sine is one-to-one.

If we restrict the domain of sine to , we obtain the one-to-one function whose graph is shown here. The inverse of this function is called the arcsine, which is abbreviated as . Here is the formal definition.

Arcsine

Suppose .

* The arcsine of t, denoted , is the angle in  whose sine equals t.

Short version:

*  means  and .

In defining , we must restrict t to be in the interval  because otherwise there is no angle whose sine equals t.

Example 5

(a) Evaluate .

(b) Evaluate .

(c) Explain why the expression  makes no sense.

Solution

(a) We have , because  and 0 is in the interval .

(b) We have , because  and  is in .

(c) The expression  makes no sense because there is no angle whose sine equals .

Do not confuse  with . The same comments that were made earlier about the notation  apply to . Specifically,  means , but  involves an inverse function.

The next example should help solidify your understanding of the arcsine function.

Example 6

Sketch the radius of the unit circle corresponding to the angle .

Solution

We seek an angle in  whose sine equals 0.3. This means that the second coordinate of the endpoint of the corresponding radius will equal 0.3. Thus we start with 0.3 on the vertical axis, as shown here, and extend a line to the right until it intersects the unit circle. That point of intersection is the endpoint of the radius corresponding to the angle , as shown here.

A calculator shows that .

Thus the angle  shown in the figure here is approximately 0.3047 radians, which is approximately .

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Because the inverse of a function interchanges the domain and range of the original function, we have the following results.

Domain and range of arcsine

* The domain of  is .
* The range of  is .

The graph of  can be obtained in the usual way when dealing with inverse functions. Specifically, flip the graph of the sine (restricted to the interval ) across the line with slope 1 that contains the origin, getting the graph shown here (see figure).

Given the lengths of the hypotenuse and another side of a right triangle, you can use the arcsine function to determine the angle opposite the nonhypotenuse side. The next example illustrates the procedure.

Example 7

Suppose your altitude goes up by 150 feet when driving one-half mile on a straight road. What is the angle of elevation of the road?

Solution

The first step toward solving a problem like this is to sketch the situation. Thus we begin by constructing the sketch shown here, which is not drawn to scale. We want to find the angle of elevation of the road; this angle has been denoted by 0 in the sketch.

As usual, we must use consistent units throughout a problem. The information we have been given uses both feet and miles. Thus we convert one-half mile to feet: because one mile equals 5280 feet, one-half mile equals 2640 feet.

Because the sine of an angle in a right triangle equals the length of the opposite side divided by the length of the hypotenuse, the sketch shows that .

Thus .

Hence  is approximately 0.057 radians, which is approximately .

The next example shows how to find the angles in an isosceles triangle, given the lengths of the sides.

Example 8

Find the angle between the two sides of length 7 in an isosceles triangle that has one side of length 8 and two sides of length 7.

Solution

Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure here (see figure).

Let  denote the angle between the perpendicular and a side of length 7. Because the base of the isosceles triangle has length 8, the side of the right triangle opposite the angle  has length 4. Thus  and hence .

The angle between the two sides of length 7 is , which equals , which is approximately 1.2165 radians, which is approximately .

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The Arctangent Function

Now we consider the tangent function, whose graph is shown below (see figure).

We need to restrict the domain to obtain a one-to-one function. We again start by declaring that 0 should be in the domain of the restricted function. Looking at the graph above of tangent, we see that  is the largest interval containing 0 on which tangent is one-to-one. Recall that the tangent function is not defined at  or at ; thus these numbers cannot be included in the domain.

If we restrict the domain of tangent to , we obtain a one-to-one function. The inverse of this function is called the arctangent, which is abbreviated as .

Arctangent

Suppose t is a real number.

* The arctangent of t, denoted  =, is the angle in  whose tangent equals t.

Short version:

*  means  and .

Example 9

(a) Evaluate .

(b) Evaluate .

(c) Evaluate .

Solution

(a) We have , because  and 0 is in the interval .

(b) We have , because  and  is in the interval .

(c) We have , because tan  and  is in the interval .

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Do not confuse  with . The same comments that were made earlier about the notation  and  apply to . Specifically,  means , but  involves an inverse function.

Example 10

Sketch the radius of the unit circle corresponding to the angle .

Solution

We seek an angle in  whose tangent equals . This means that the slope of the corresponding radius will equal . The unit circle has two radii with slope ; one of them is the radius shown here (see figure) and the other is the radius in the opposite direction. But of these two radii, only the one shown here has a corresponding angle in the interval . Notice that the indicated angle is negative because of the clockwise direction of the arrow.

A calculator shows that .

Thus the angle  shown here is approximately  radians, which is approximately .

Unlike  and , which make sense only when t is in ,  makes sense for every real number t (because for every real number t there is an angle whose tangent equals t). Because the inverse of a function interchanges the domain and range of the original function, the domain of the arctangent is the set of real numbers and the range of the arctangent is the interval .

The table below summarizes the domain and range of all three inverse trigonometric functions. Here the set of real numbers is denoted using the interval notation .

Domain and range of the inverse trigonometric functions

|  | **Domain** | **Range** |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |

The graph of  can be obtained in the usual way when dealing with inverse functions. Specifically, flip the graph of the tangent (restricted to an interval slightly smaller than ) across the line with slope 1 that contains the origin, getting the graph shown below (see figure).

Given the lengths of the two nonhypotenuse sides of a right triangle, you can use the arctangent function to determine the angles of the triangle. The next example illustrates the procedure.

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Example 11

(a) In this right triangle (see figure), use the arctangent function to evaluate the angle u.

(b) In this right triangle, use the arctangent function to evaluate the angle v.

(c) As a check, compute the sum of the angles u and v obtained in parts (a) and (b). Does this sum have the expected value?

Solution

(a) Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have . Thus .

Hence u is approximately 0.5071 radians, which is approximately .

(b) Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have . Thus .

Hence v is approximately 1.0637 radians, which is approximately .

(c) We have .

Thus the sum of the two acute angles in this right triangle is , as expected.

Given the slope of a line, the arctangent function allows us to find the angle the line makes with the positive horizontal axis, as shown in the next example.

Example 12

What angle does the line  in the xy-plane make with the positive x-axis?

Solution

We seek the angle  shown in the figure here (see figure). Because the line  has slope , we have .

Thus .

Hence  is approximately 0.588 radians, which is approximately .

Exercises

You should be able to do Exercises 1–4 without a calculator.

1 Evaluate .

2 Evaluate .

3 Evaluate .

4 Evaluate .

Exercises 5–16 emphasize the importance of understanding inverse notation as well as the importance of parentheses in determining the order of operations.

5 For x = 0.3, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

6 For x = 0.4, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

7 For , evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

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8 For , evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

9 For x = 2, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

10 For x = 3, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

11 For x = 4, evaluate each of the following:

(a) 

(b) 

(c) 

12 For x = 5, evaluate each of the following:

(a) 

(b) 

(c) 

13 For x = 6, evaluate each of the following:

(a) 

(b) 

(c) 

14 For x = 9, evaluate each of the following:

(a) 

(b) 

(c) 

15 For x = 0.1, evaluate each of the following:

(a) 

(b) 

(c) 

16 For x = 0.2, evaluate each of the following:

(a) 

(b) 

(c) 

Use the right triangle above (see figure) for Exercises 17–24. This triangle is not drawn to scale corresponding to the data in the exercises.

17 Suppose a = 2 and c = 3. Evaluate u in radians.

18 Suppose a = 3 and c = 4. Evaluate u in radians.

19 Suppose a = 2 and c = 5. Evaluate v in radians.

20 Suppose a = 3 and c = 5. Evaluate v in radians.

21 Suppose a = 5 and b = 4. Evaluate u in degrees.

22 Suppose a = 5 and b = 6. Evaluate u in degrees.

23 Suppose a = 5 and b = 7. Evaluate v in degrees.

24 Suppose a = 7 and b = 6. Evaluate v in degrees.

25 Find the angle between the two sides of length 9 in an isosceles triangle that has one side of length 14 and two sides of length 9.

26 Find the angle between the two sides of length 8 in an isosceles triangle that has one side of length 7 and two sides of length 8.

27 Find the angle between a side of length 6 and the side with length 10 in an isosceles triangle that has one side of length 10 and two sides of length 6.

28 Find the angle between a side of length 5 and the side with length 9 in an isosceles triangle that has one side of length 9 and two sides of length 5.

29 Find the smallest positive number  such that .

30 Find the smallest positive number  such that .

31 Find the smallest positive number  such that .

32 Find the smallest positive number  such that .

33 Find the second smallest positive number  such that .

34 Find the second smallest positive number  such that .

35 Find the smallest positive number y such that .

36 Find the smallest positive number y such that .

37 Find the smallest positive number x such that.

38 Find the smallest positive number x such that.

39 Find the smallest positive number x such that.

40 Find the smallest positive number x such that.

41 Find the smallest positive number  such that . [Hint: Careful, the answer is not .]

42 Find the smallest positive number  such that  . [Hint: Careful, the answer is not .]

43 Find the smallest number  larger than  such that .

44 Find the smallest number  larger than  such that .

45 What angle does the line  in the xy-plane make with the positive x-axis?

46 What angle does the line  in the xy-plane make with the positive x-axis?

47 What is the angle between the positive horizontal axis and the line containing the points (3, 1) and (5, 4)?

48 What is the angle between the positive horizontal axis and the line containing the points (2, 5) and (6, 2)?

For Exercises 49–52: Hilly areas often have road signs giving the percentage grade for the road. A 5% grade, for example, means that the altitude changes by 5 feet for each 100 feet of horizontal distance.

49 What percentage grade should be put on a road sign where the angle of elevation of the road is ?

50 What percentage grade should be put on a road sign where the angle of elevation of the road is ?

51 Suppose an uphill road sign indicates a road grade of 6%. What is the angle of elevation of the road?

52 Suppose an uphill road sign indicates a road grade of 8%. What is the angle of elevation of the road?

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Problems

Some problems require considerably more thought than the exercises.

53 Without using a calculator, sketch the unit circle and the radius corresponding to .

54 Without using a calculator, sketch the unit circle and the radius corresponding to .

55 Without using a calculator, sketch the unit circle and the radius corresponding to .

56 Explain why the expression  makes no sense.

57 Explain why . [Hint: Take a = 3 and b = 4 in the triangle used for Exercises 17–24. Then find c and consider various ways to express u.]

58 Explain why .

59 Suppose a and b are numbers such that . Explain why .

60 Find all numbers t such that .

61 There exist angles  such that  (for example,  and  are two such angles). However, explain why there do not exist any numbers t such that .

62 Show that in an isosceles triangle with two sides of length b and a side of length c, the angle between the two sides of length b is  .

63 Show that in an isosceles triangle with two sides of length b and a side of length c, the angle between a side of length b and the side of length c is .

64 Suppose you are asked to find the angles in an isosceles triangle that has two sides of length 5 and one side of length 11. Using the two previous problems, this would require you to compute  and  , neither of which makes sense. What is wrong here?

Worked-Out Solutions to Odd-Numbered Exercises

Do not read these worked-out solutions before attempting to do the exercises yourself. Otherwise you may mimic the techniques shown here without understanding the ideas.

Best way to learn: Carefully read the section of the textbook, then do all the odd-numbered exercises and check your answers here. If you get stuck on an exercise, then look at the worked-out solution here.

You should be able to do Exercises 1–4 without a calculator.

1 Evaluate .

Solution

; thus.

3 Evaluate .

Solution

; thus .

Exercises 5–16 emphasize the importance of understanding inverse notation as well as the importance of parentheses in determining the order of operations.

5 For x = 0.3, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

Solution

(a) 

(b) 

(c) 

(d) 

7 For , evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

Solution

(a) 

(b) 

(c) 

(d) 

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9 For x = 2, evaluate each of the following:

(a) 

(b) 

(c) 

(d) 

Solution

(a) 

(b) 

(c) 

(d) 

11 For x = 4, evaluate each of the following:

(a) 

(b) 

(c) 

Solution

(a) 

(b) 

(c) 

13 For x = 6, evaluate each of the following:

(a) 

(b) 

(c) 

Solution

(a) 

(b) 

(c) 

15 For x = 0.1, evaluate each of the following:

(a) 

(b) 

(c) 

Solution

(a) 

(b) 

(c) 

Use the right triangle above (see figure) for Exercises 17–24. This triangle is not drawn to scale corresponding to the data in the exercises.

17 Suppose a = 2 and c = 3. Evaluate u in radians.

Solution

Because the cosine of an angle in a right triangle equals the length of the adjacent side divided by the length of the hypotenuse, we have . Using a calculator working in radians, we then have  radians.

19 Suppose a = 2 and c = 5. Evaluate v in radians.

Solution

Because the sine of an angle in a right triangle equals the length of the opposite side divided by the length of the hypotenuse, we have . Using a calculator working in radians, we then have  radians.

21 Suppose a = 5 and b = 4. Evaluate u in degrees.

Solution

Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have . Using a calculator working in degrees, we then have.

23 Suppose a = 5 and b = 7. Evaluate v in degrees.

Solution

Because the tangent of an angle in a right triangle equals the length of the opposite side divided by the length of the adjacent side, we have . Using a calculator working in degrees, we then have.

25 Find the angle between the two sides of length 9 in an isosceles triangle that has one side of length 14 and two sides of length 9.

Solution

Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below (see figure).

Let  denote the angle between the perpendicular and a side of length 9. Because the base of the isosceles triangle has length 14, the side of the right triangle opposite the angle  has length 7. Thus . Hence .

Thus the angle between the two sides of length 9 is approximately 1.7822 radians (), which is approximately .

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27 Find the angle between a side of length 6 and the side with length 10 in an isosceles triangle that has one side of length 10 and two sides of length 6.

Solution

Create a right triangle by dropping a perpendicular from the vertex to the base, as shown in the figure below (see figure).

Let  denote the angle between the side of length 10 and a side of length 6. Because the base of the isosceles triangle has length 10, the side of the right triangle adjacent to the angle  has length 5. Thus . Hence .

Thus the angle between a side of length 6 and the side with length 10 is approximately 0.58569 radians, which is approximately .

29 Find the smallest positive number  such that .

Solution

The equation above implies that . Thus we take .

31 Find the smallest positive number  such that .

Solution

The equation above implies that . Thus we take .

33 Find the second smallest positive number  such that .

Solution

Take the log of both sides of the equation above, getting, which implies that .

The smallest positive number  satisfying this equation is . The second smallest positive number satisfying this equation is .

35 Find the smallest positive number y such that .

Solution

The equation above implies that we should choose .Thus we should choose .

37 Find the smallest positive number x such that.

Solution

Write . Then the equation above can be rewritten as .

Using the quadratic formula, we find that the solutions to this equation are  and .

Thus  or . However, there is no real number x such that  (because  is at most 1 for every real number x), and thus we must have . Thus .

39 Find the smallest positive number x such that.

Solution

Write . Then the equation above can be rewritten as .

Using the quadratic formula or factorization, we find that the solutions to this equation are  and .

Thus  or , which suggests that we choose  or . Because arccosine is a decreasing function,  is smaller than . Because we want to find the smallest positive value of x satisfying the original equation, we choose .

41 Find the smallest positive number  such that . [Hint: Careful, the answer is not .]

Solution

The answer is not  because  is a negative number and we need to find the smallest positive number  such that . In the figure below (see figure), the orange radius corresponds to the negative angle . The arrow and the blue radius show the angle we seek, which is , which is approximately 3.55311 radians (which is approximately ).

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43 Find the smallest number  larger than  such that .

Solution

The desired number is, which is approximately 7.07858.

45 What angle does the line  in the xy-plane make with the positive x-axis?

Solution

We seek the angle  shown in the figure here (see figure). Because the line  has slope , we have . Thus .

Hence  is approximately 0.3805 radians, which is approximately .

47 What is the angle between the positive horizontal axis and the line containing the points (3, 1) and (5, 4)?

Solution

Let  denote the angle between the positive horizontal axis and the line containing (3, 1) and (5, 4).

The line containing (3, 1) and (5, 4) has slope , which equals . Thus . Thus . Hence  is approximately 0.982794 radians, which is approximately .

For Exercises 49–52: Hilly areas often have road signs giving the percentage grade for the road. A 5% grade, for example, means that the altitude changes by 5 feet for each 100 feet of horizontal distance.

49 What percentage grade should be put on a road sign where the angle of elevation of the road is ?

Solution

The grade of a portion of a road is the change in altitude divided by the change in horizontal distance. Thus the grade is the slope of the road. A road with a  angle of elevation has slope , which is approximately 0.052. Thus a road sign should indicate a 5% grade.

51 Suppose an uphill road sign indicates a road grade of 6%. What is the angle of elevation of the road?

Solution

Let  denote the angle of elevation of the road. A road grade of 6% means that . Thus. Hence  is approximately 0.0599 radians, which is approximately .